MA261 Quiz 6

July 12, 2016

Problem 1.

Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^2 \, dy \, dx$$

Solution.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^2 \, dy \, dx$$

$$= \int_0^1 \int_0^{\pi/2} r^7 \sin^2(\theta) \, dr \, d\theta$$

$$= \frac{1}{8} \cdot \frac{\pi}{4} = \frac{\pi}{32}$$

Problem 2.

Find the surface area of the part of the plane $3x - y + z = 1$ that lies inside the cylinder $(x-1)^2 + y^2 = 1$

Solution.

$$z = f(x,y) = 1 - 3x + y$$
$$f_x = -3$$
$$f_y = 1$$

So the area is

$$\int_{(x-1)^2+y^2 \le 1} \sqrt{(-3)^2 + (1)^2 + 1} \, dA$$

$$= \sqrt{11} \int_{(x-1)^2+y^2 \le 1} dA = \sqrt{11}\pi$$